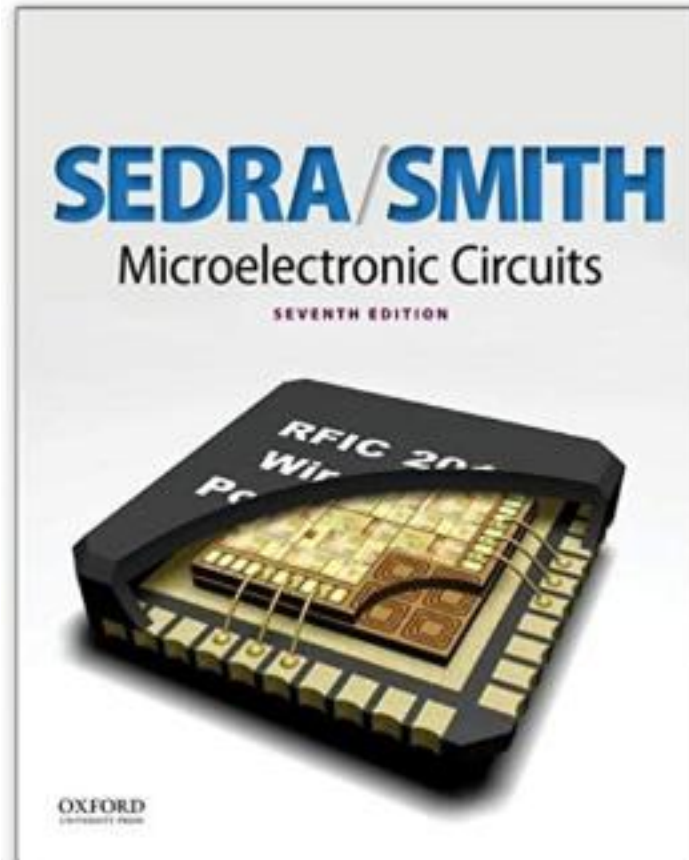


Operação Física dos Diodos

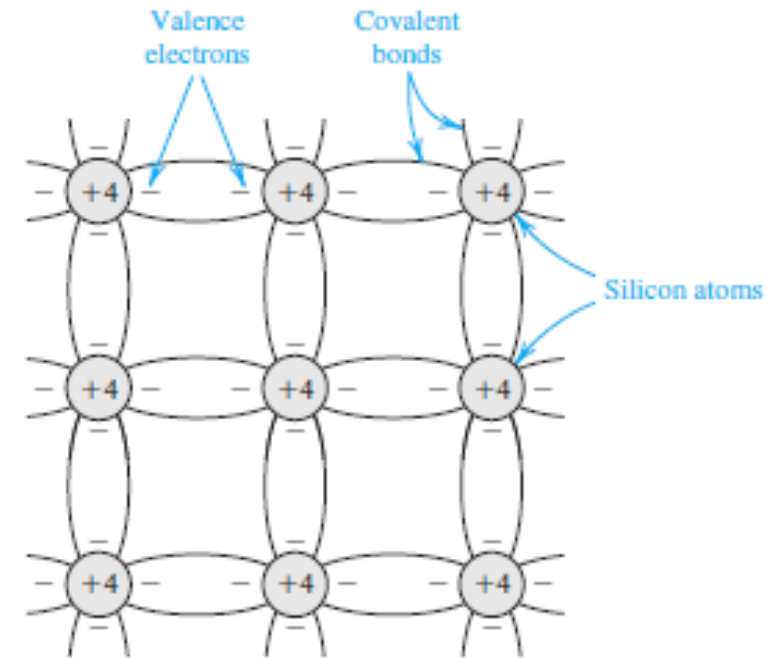


Chapter 3
Diodes

Intrinsic Semiconductors

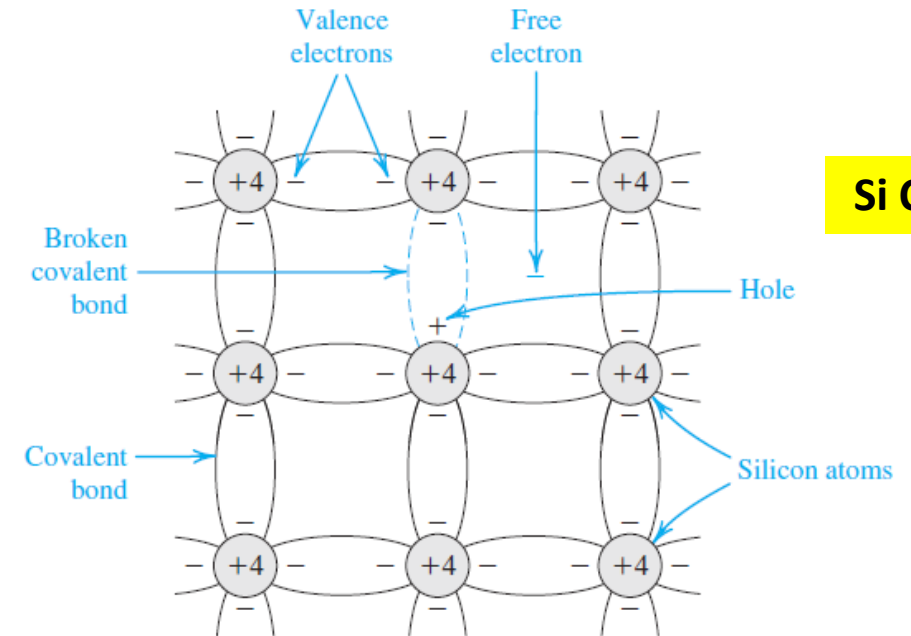
A silicon atom has four valence electrons, and thus it requires another four to complete its outermost shell. This is achieved by sharing one of its valence electrons with each of its four neighboring atoms. Each pair of shared electrons forms a covalent bond. The result is that a crystal of pure or intrinsic silicon has a regular lattice structure, where the atoms are held in their position by the covalent bonds. Figure shows a two-dimensional representation of such a structure.

Si Crystal



Two-dimensional representation of the silicon crystal. The circles represent the inner core of silicon atoms, with +4 indicating its positive charge of $+4q$, which is neutralized by the charge of the four valence electrons. Observe how the covalent bonds are formed by sharing of the valence electrons. At 0 K, all bonds are intact and no free electrons are available for current conduction.

At room temperature, sufficient thermal energy exists to break some of the covalent bonds, a process known as **thermal generation**. As shown in figure, when a covalent bond is broken, an electron is freed. The **free electron** can wander away from its parent atom, and it **becomes available to conduct electric current if an electric field is applied to the crystal**. As the electron leaves its parent atom, it **leaves behind a net positive charge, equal to the magnitude of the electron charge**. Thus, an **electron from a neighboring atom may be attracted to this positive charge, and leaves its parent atom**. This action fills up the “hole” that existed in the ionized atom but creates a new hole in the other atom. This process may repeat itself, with the result that we effectively have a positively charged carrier, or **hole**, moving through the silicon crystal structure and being available to conduct electric current. The charge of a hole is equal in magnitude to the charge of an electron. We can thus see that **as temperature increases, more covalent bonds are broken and electron-hole pairs are generated**. The increase in the numbers of free electrons and holes results in an increase in the conductivity of silicon.



At room temperature, some of the covalent bonds are broken by thermal generation. Each broken bond gives rise to a free electron and a hole, both of which become available for current conduction.

- **Thermal generation results in free electrons and holes in equal numbers and hence equal concentrations**, where concentration refers to the number of charge carriers per unit volume (cm^3). **The free electrons and holes move randomly through the silicon crystal structure, and in the process some electrons may fill some of the holes.** This process, called **recombination**, results in the disappearance of free electrons and holes.

The recombination rate is proportional to the number of free electrons and holes, which in turn is determined by the thermal generation rate. The latter is a strong function of temperature. **In thermal equilibrium, the recombination rate is equal to the generation rate**, and one can conclude that the concentration of free electrons n is equal to the concentration of holes p ,

$$n = p = n_i \quad (3.1)$$

n_i denotes the number of free electrons and holes in a unit volume (cm^3) of intrinsic silicon at a given temperature.

Results from semiconductor physics gives n_i as:

$$n_i = BT^{3/2} e^{-E_g/2kT} \quad (3.2)$$

$k = 8,62 \times 10^{-5}$ eV/K (Boltzman's constant)

For Si:

$B = 7,3 \times 10^{15}$ cm⁻³ K^{-3/2}

$E_g = 1,12$ eV

E_g is the minimum energy required to break a covalent bond and thus generate an electron hole pair.

Example 3.1

Calculate the value of n_i for silicon at room temperature ($T \approx 300$ K).

Solution

Substituting the values given above in Eq. (3.2) provides

$$\begin{aligned}n_i &= 7.3 \times 10^{15} (300)^{3/2} e^{-1.12 / (2 \times 8.62 \times 10^{-5} \times 300)} \\ &= 1.5 \times 10^{10} \text{ carriers/cm}^3\end{aligned}$$

Although this number seems large, to place it into context note that silicon has 5×10^{22} atoms/cm³. Thus at room temperature only one in about 5×10^{12} atoms is ionized and contributing a free electron and a hole!

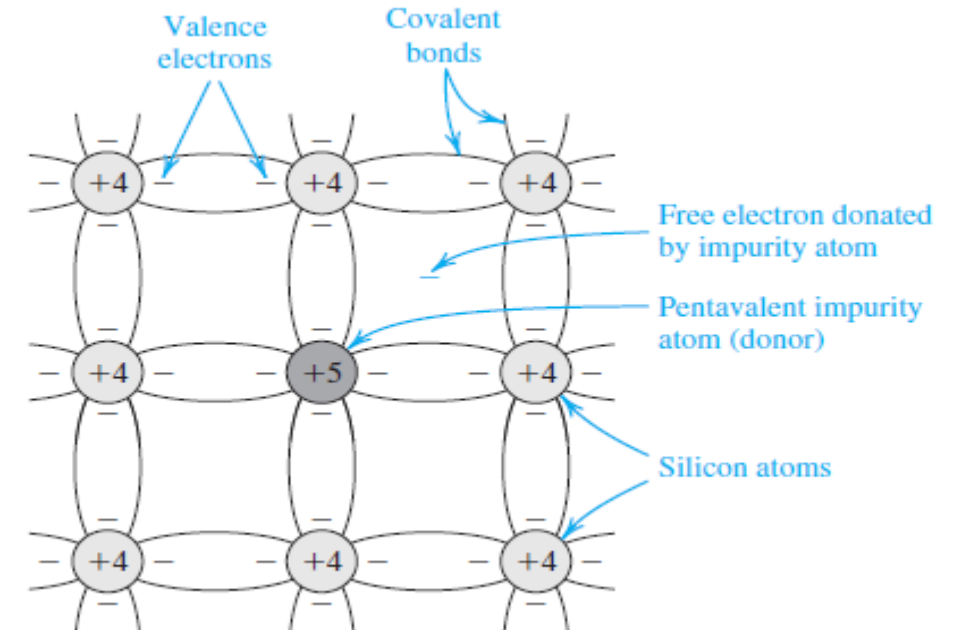
Doped Semiconductors

n-type material

Doping involves introducing impurity atoms into the silicon crystal in sufficient numbers to substantially increase the concentration of either free electrons or holes but with little or no change in the crystal properties of silicon. To increase the concentration of free electrons, **n**, silicon is doped with an element with a valence of 5, such as phosphorus.

Thus each phosphorus atom *donates* a free electron to the silicon crystal, and the phosphorus impurity is called a **donor**.

Doped Si Crystal



A silicon crystal doped by a pentavalent element. Each dopant atom donates a free electron and is thus called a donor. The doped semiconductor becomes **n type**.

If the **concentration of donor atoms is N_D** , where is usually much greater than n_i , the **concentration of free electrons in the n -type (n_n)** silicon will be:

$$n_n \approx N_D$$

The **concentration of holes in the n -type (p_n)** can be find using the following relationship:

$$p_n n_n = n_i^2 \quad (3.3) \quad \longrightarrow \quad p_n \approx \frac{n_i^2}{N_D}$$

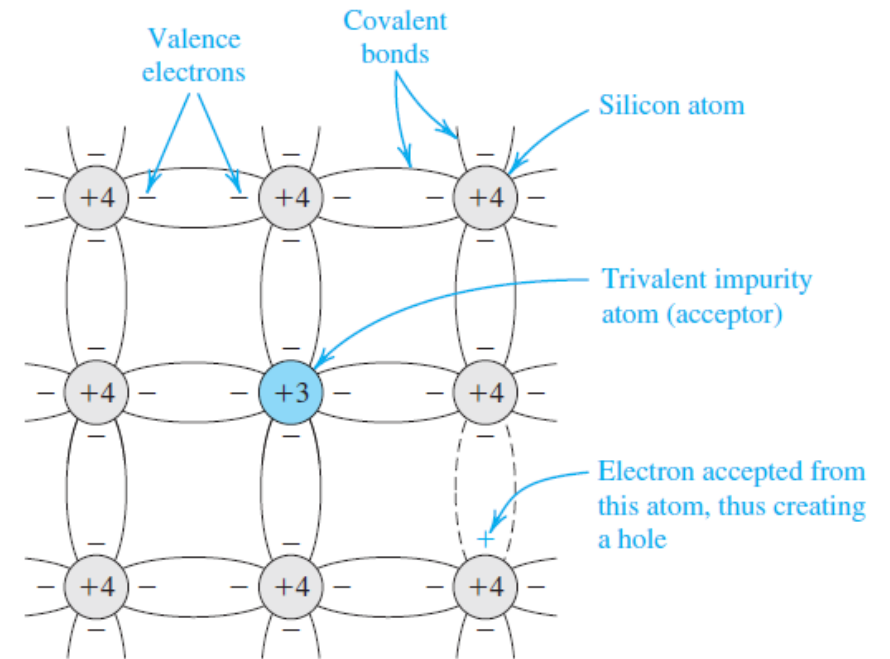
We note that **in n -type silicon the concentration of free electrons will be much larger than that of holes**. Hence electrons are said to be the **majority** charge carriers and holes the **minority** charge carriers in n -type silicon.

p-type material

To obtain **p-type silicon** in which holes are the majority charge carriers, **a trivalent impurity such as boron is used**. Figure shows a silicon crystal doped with boron. Note that the boron atoms replace some of the silicon atoms in the silicon crystal structure.

Since each boron atom has three electrons in its outer shell, it **accepts** an electron from a neighboring atom, thus forming covalent bonds. The result is a hole in the neighboring atom and a bound negative charge at the **acceptor** (boron) atom. It follows that each acceptor atom provides a hole.

Doped Si Crystal



A silicon crystal doped with a trivalent impurity. **Each dopant atom gives rise to a hole**, and the semiconductor becomes **p type**.

If the acceptor doping concentration is N_A , where is usually much greater than n_i , the hole concentration (p_p) becomes:

$$p_p \approx N_A$$

The concentration of minority electrons can be found by using the relationship:

$$p_p n_p = n_i^2 \quad \longrightarrow \quad n_p \approx \frac{n_i^2}{N_A}$$

It should be emphasized that a piece of n -type or p -type silicon is electrically neutral: the charge of the majority free carriers (electrons in the n -type and holes in the p -type silicon) are neutralized by the bound charges associated with the impurity atoms.

Example 3.2

Consider an n -type silicon for which the dopant concentration $N_D = 10^{17}/\text{cm}^3$. Find the electron and hole concentrations at $T = 300$ K.

Solution

The concentration of the majority electrons is

$$n_n \approx N_D = 10^{17}/\text{cm}^3$$

The concentration of the minority holes is

$$p_n \approx \frac{n_i^2}{N_D}$$

In Example 3.1 we found that at $T = 300$ K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$. Thus,

$$\begin{aligned} p_n &= \frac{(1.5 \times 10^{10})^2}{10^{17}} \\ &= 2.25 \times 10^3/\text{cm}^3 \end{aligned}$$

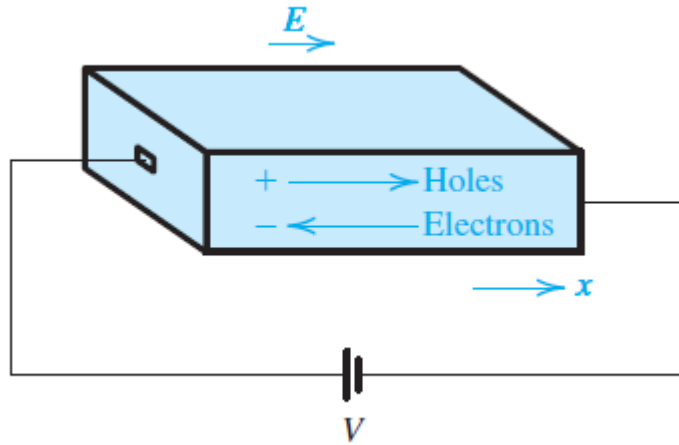


Observe that $n_n \gg n_i$ and that n_n is vastly higher than p_n .

Current Flow in Semiconductors

- There are two distinctly different mechanisms for the movement of charge carriers and hence for current flow in semiconductors: **drift** and **diffusion**.

Drift Current



An electric field E established in a bar of silicon causes the holes to drift in the direction of E and the free electrons to drift in the opposite direction. Both the hole and electron drift currents are in the direction of E .

- The holes acquire a velocity $v_{p\text{-drift}}$ given by:

$$v_{p\text{-drift}} = \mu_p E \quad (3.8)$$

μ_p is a constant called the **hole mobility**: It represents the degree of ease by which holes move through the silicon crystal in response to the electrical field E . For intrinsic silicon $\mu_p = 480 \text{ cm}^2/\text{V.s}$.

Current density of holes (J_p) and electron (J_n)

Let the concentration of holes be p and that of free electrons n . Consider a plane perpendicular to the x direction, sendo A a área da seção transversal e q a carga do eletron. In one second, the hole charge that crosses that plane will be:

$$Aqp v_{p\text{-drift}} \longrightarrow I_p = Aqp v_{p\text{-drift}} \longrightarrow I_p = Aqp \mu_p E \quad (3.10)$$

The current density (J_p) is:

$$J_p = \frac{I_p}{A} = qp \mu_p E \quad (3.11)$$

- The free electrons acquire a velocity $v_{p\text{-drift}}$ given by:

$$v_{n\text{-drift}} = -\mu_n E$$

μ_n is a constant called the **electron mobility**: It represents the degree of ease by which holes move through the silicon crystal in response to the electrical field E . For intrinsic silicon $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$.

- The current component due to the drift of free electrons can be found in a similar manner.

$$I_n = -Aqn v_{n\text{-drift}} \longrightarrow J_n = qn\mu_n E$$

■ The total drift current density can now be found by summing of J_p and J_n :

$$J = J_p + J_n = q(p\mu_p + n\mu_n)E \longrightarrow J = \sigma E \quad (\sigma \text{ is the conductivity})$$

$$\sigma = q(p\mu_p + n\mu_n)$$

$$\longrightarrow J = E/\rho \quad \longrightarrow \rho \equiv \frac{1}{\sigma} = \frac{1}{q(p\mu_p + n\mu_n)}$$

(ρ is the resistivity)

Example 3.3

Find the resistivity of (a) intrinsic silicon and (b) p -type silicon with $N_A = 10^{16}/\text{cm}^3$. Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$, and assume that for intrinsic silicon $\mu_n = 1350 \text{ cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 480 \text{ cm}^2/\text{V}\cdot\text{s}$, and for the doped silicon $\mu_n = 1110 \text{ cm}^2/\text{V}\cdot\text{s}$ and $\mu_p = 400 \text{ cm}^2/\text{V}\cdot\text{s}$. (Note that doping results in reduced carrier mobilities).

Solution

(a) For intrinsic silicon,

$$p = n = n_i = 1.5 \times 10^{10}/\text{cm}^3$$

Thus,

$$\rho = \frac{1}{q(p\mu_p + n\mu_n)}$$

$$\begin{aligned}\rho &= \frac{1}{1.6 \times 10^{-19} (1.5 \times 10^{10} \times 480 + 1.5 \times 10^{10} \times 1350)} \\ &= 2.28 \times 10^5 \text{ } \Omega \cdot \text{cm}\end{aligned}$$

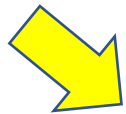
(b) For the *p*-type silicon

$$p_p \approx N_A = 10^{16} / \text{cm}^3$$

$$n_p \approx \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 / \text{cm}^3$$

Thus,

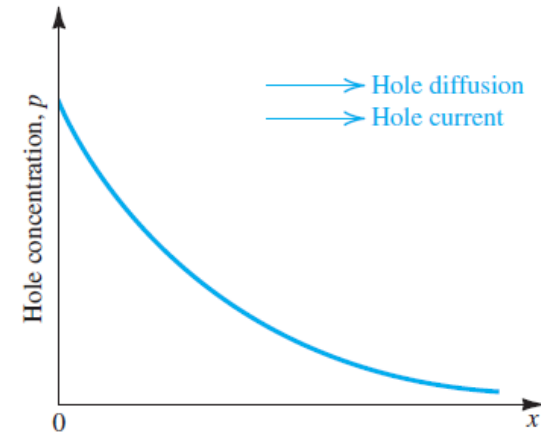
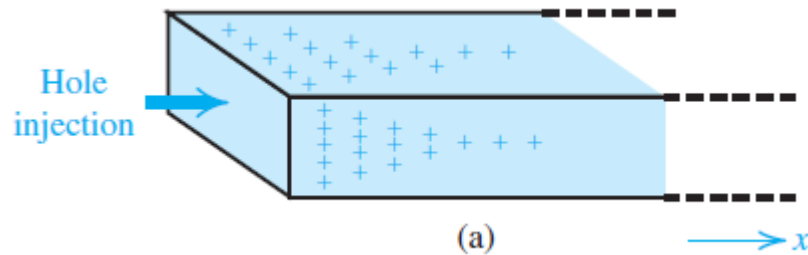
$$\begin{aligned} \rho &= \frac{1}{q(p\mu_p + n\mu_n)} \\ &= \frac{1}{1.6 \times 10^{-19} (10^{16} \times 400 + 2.25 \times 10^4 \times 1110)} \\ &\approx \frac{1}{1.6 \times 10^{-19} \times 10^{16} \times 400} = 1.56 \text{ } \Omega \cdot \text{cm} \end{aligned}$$



Observe that doping the silicon reduces its resistivity by a factor of about 10^4 , a truly remarkable change !

Diffusion Current

Carrier diffusion occurs when the density of charge carriers in a piece of semiconductor is not uniform. For instance, if by some mechanism the concentration of, say, holes, is made higher in one part of a piece of silicon than in another, then holes will diffuse from the region of high concentration to the region of low concentration.



The magnitude of the current at any point is proportional to the slope of the concentration profile, or the concentration gradient, at that point.

$$J_p = -qD_p \frac{dp(x)}{dx}$$

J_p is the **hole-current density** (A/cm²)

D_p is a constant called the **diffusion constant** or **diffusivity** of holes

For Si $D_p = 12 \text{ cm}^2/\text{s}$

- In the case of electron diffusion resulting from an electron concentration gradient, a similar relationship applies:

$$J_n = qD_n \frac{dn(x)}{dx}$$

J_p is the **hole-current density** (A/cm²)

D_n is a constant called the **diffusion constant** or **diffusivity** of electrons

For Si $D_n = 35 \text{ cm}^2/\text{s}$

Relationship between D and μ

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

(Einstein Relationship)

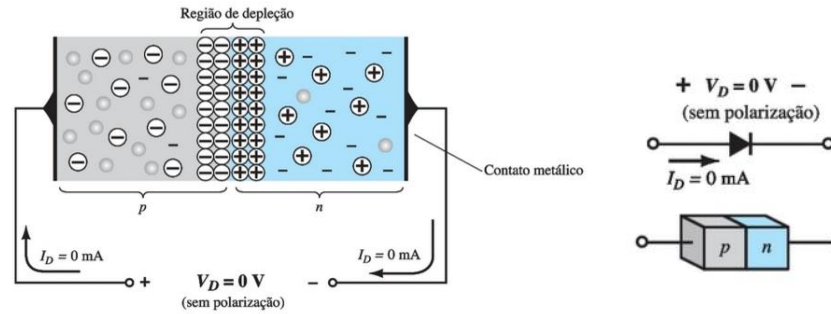
$$V_T = kT/q \longrightarrow V_T \approx 25.9 \text{ mV at } 27^\circ\text{C.}$$

(thermal voltage)

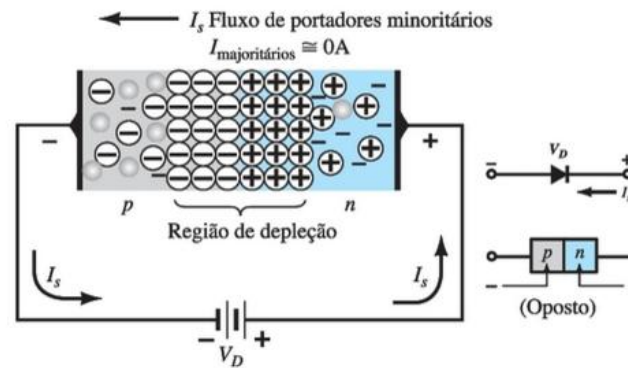
**pn junction
in equilibrium,
forward biased and
reverse biased**

A diode has three operating conditions:

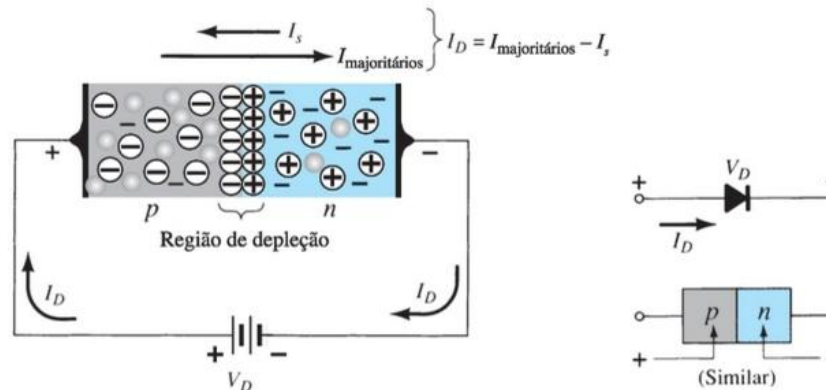
No Applied Bias ($V_D=0$)



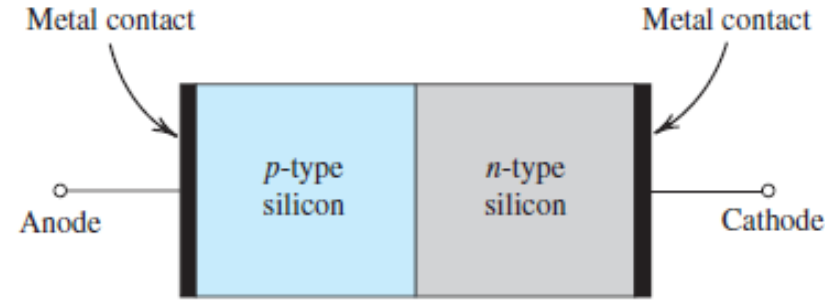
Reverse-Bias ($V_D < 0$)



Forward-Bias ($V_D > 0$)



pn junction in equilibrium

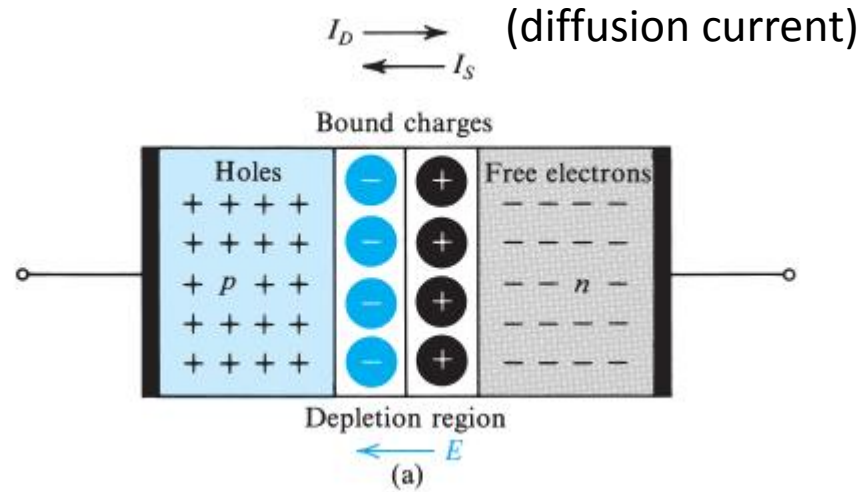


The charge of these holes is neutralized by an equal amount of bound negative charge associated with the acceptor atoms.

The minority electrons are generated in the *p*-type material by thermal ionization.

The bound positive charge neutralizes the charge of the majority electrons.

The minority holes are generated by thermal ionization but

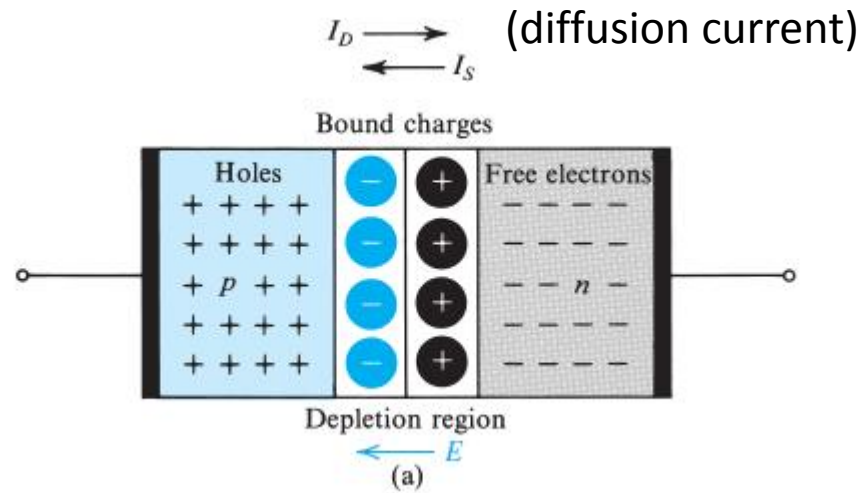


Because the concentration of holes is high in the p region and low in the n region, **holes diffuse across the junction from the p side to the n side**. Similarly, **electrons diffuse across the junction from the n side to the p side**. These two current components add together to form the **diffusion current I_D** , whose direction is from the p side to the n side.

Holes (p side)

The holes that diffuse across the junction into the n region quickly recombine with some of the majority electrons present there and thus disappear from the scene.

Since recombination takes place close to the junction, there will be a region close to the junction that is depleted of free electrons.



Electrons (n side)

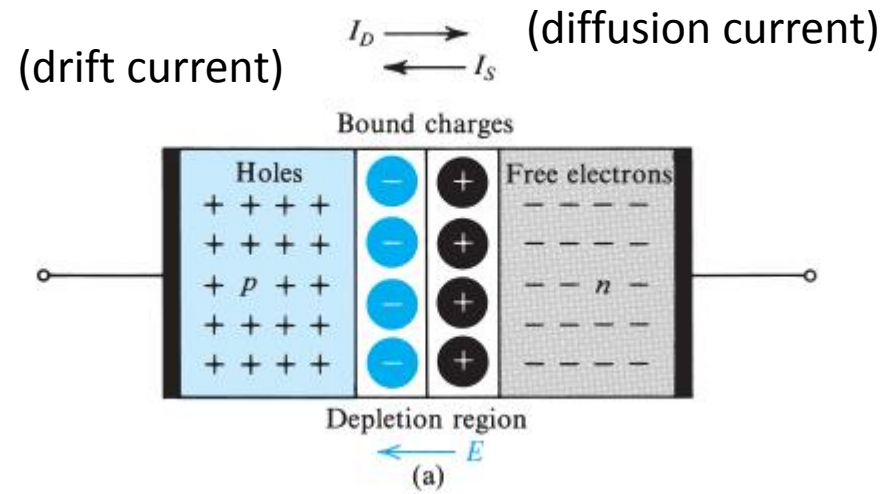
The electrons that diffuse across the junction into the p region quickly recombine with some of the majority holes there, and thus disappear from the scene.

Since recombination takes place close to the junction, in the p material close to the junction, there will be a region depleted of holes.

The charges on both sides of the depletion region cause an **electric field E to be established across the region** in the direction indicated by the figure.

Thus the resulting electric field opposes the diffusion of holes into the n region and electrons into the p region.

The voltage drop across the depletion region acts as a barrier that has to be overcome for holes to diffuse into the n region and electrons to diffuse into the p region. The



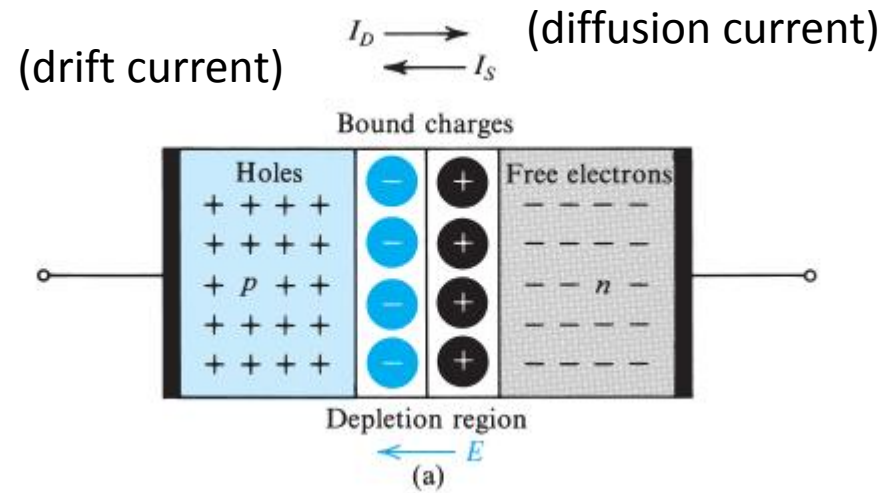
In addition to the current component I_D due to majority-carrier diffusion, **a component due to minority carrier drift exists across the junction.**

Holes (n side)

Some of the thermally generated holes in the n material move toward the junction and reach the edge of the depletion region. There, they experience the electric field in the depletion region, which sweeps them across that region into the p side.

Electrons (p side)

Some of the minority thermally generated electrons in the p material move to the edge of the depletion region and get swept by the electric field in the depletion region across that region into the n side.



Under open-circuit conditions (Fig. 3.9) no external current exists; thus the two opposite currents across the junction must be equal in magnitude:

$$I_D = I_S$$

With no external voltage applied, the barrier voltage V_0 across the pn junction can be shown to be given by:

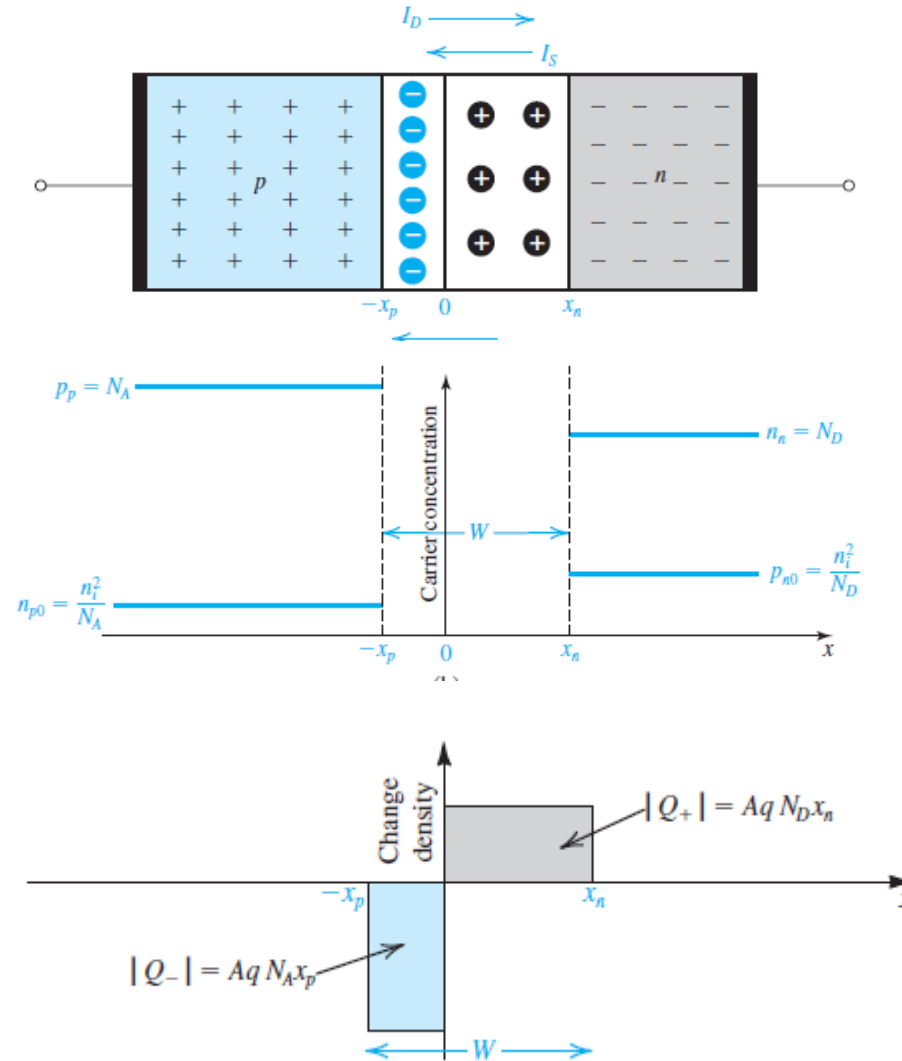
$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Typically, for silicon at room temperature, V_0 is in the range of 0.6 V to 0.9 V.

When the pn junction terminals are left open-circuited, the voltage measured between them will be zero. This is because of the contact voltages existing at the **metal semiconductor junctions at the terminals, which counter and exactly balance the barrier voltage.**

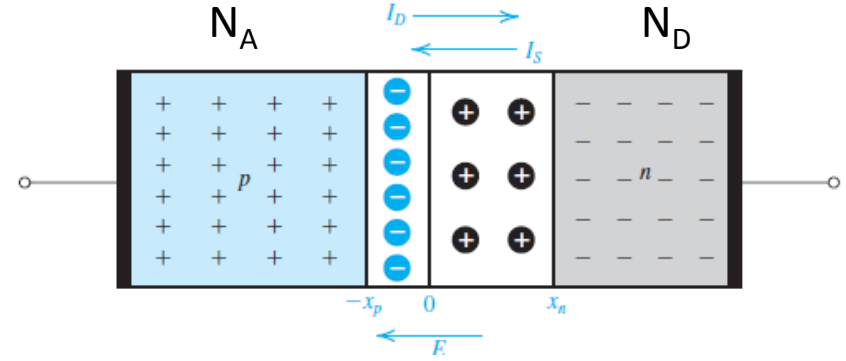
Width and Charge in the Depletion Region

The figure shows a junction in which $N_A \gg N_D$, a typical situation in practice.

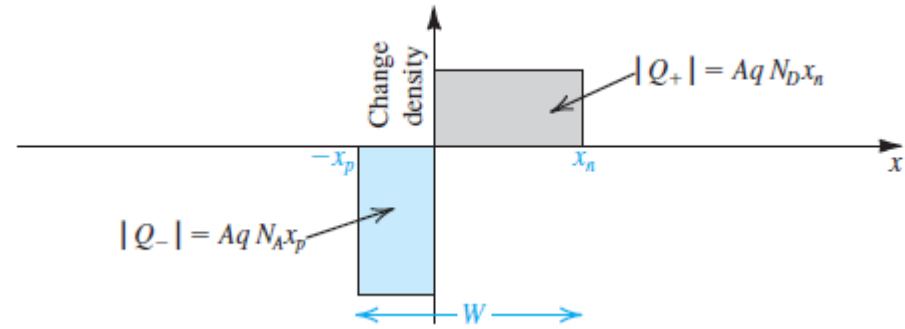
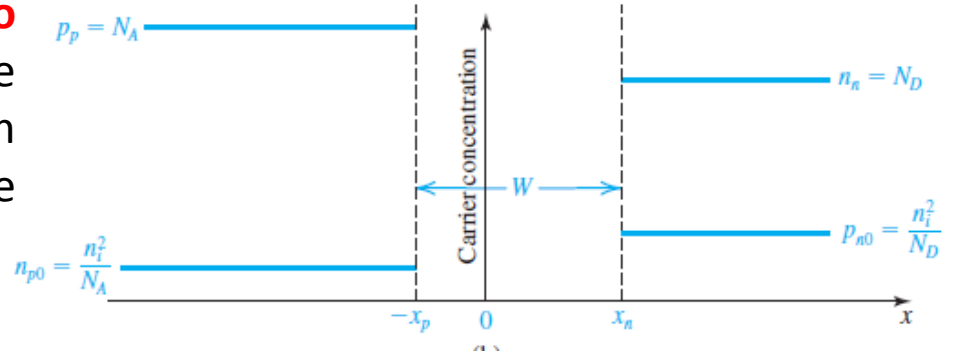


A largura da camada de depleção (W)

The figure shows a junction in which $N_A > N_D$, a typical situation in practice.

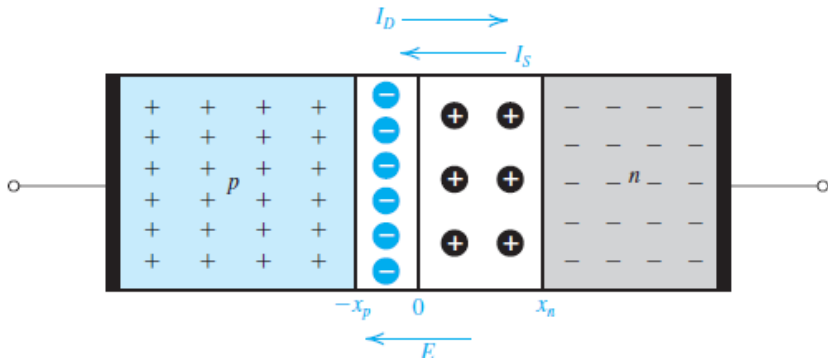


Usually unequal dopings are used. **The width of the depletion layer will not be the same on the two sides.** Rather, to uncover the same amount of charge, the depletion layer will extend deeper into the more lightly doped material.



Note that we have denoted the minority carrier concentrations in both sides by n_{p0} and p_{n0} , with the additional subscript "0" signifying equilibrium.

A largura da camada de depleção (W)



$$|Q_+| = qAx_nN_D$$

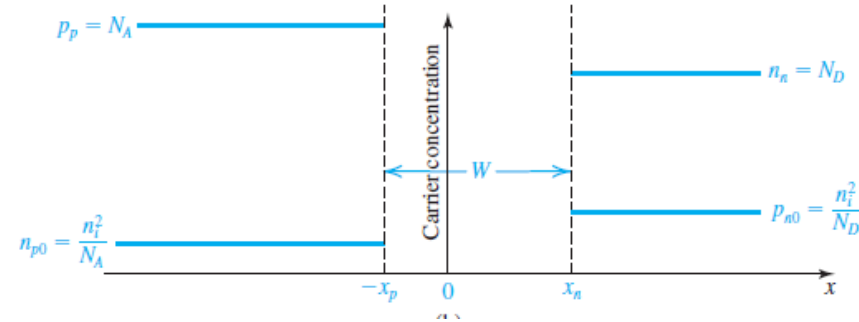
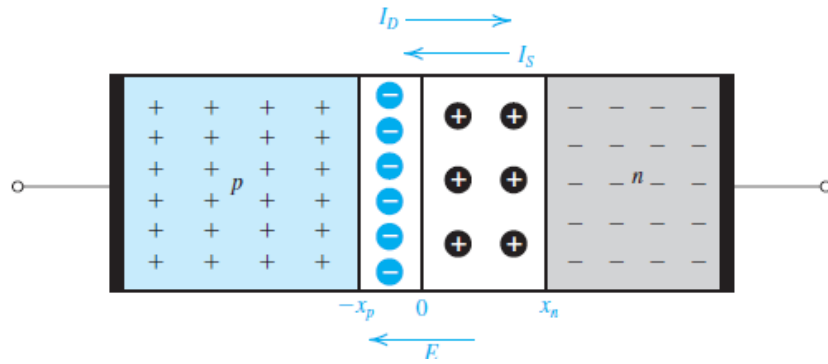
$$\longrightarrow qAx_nN_D = qAx_pN_A$$

$$|Q_-| = qAx_pN_A$$

$$\longrightarrow \frac{x_n}{x_p} = \frac{N_A}{N_D}$$

In actual practice, it is usual for one side of the junction to be much more heavily doped than the other, with the result that the depletion region exists almost entirely on one side (the lightly doped side).

A largura da camada de depleção (W)



The width W of the depletion layer can be shown to be given by:

$$W = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) V_0}$$

ϵ_s is the electrical permittivity of silicon.

$$\epsilon_s = 11.7 \epsilon_0 = 11,7 \times 8,85 \times 10^{-14} = 1.04 \times 10^{-12} \text{ F/cm}$$

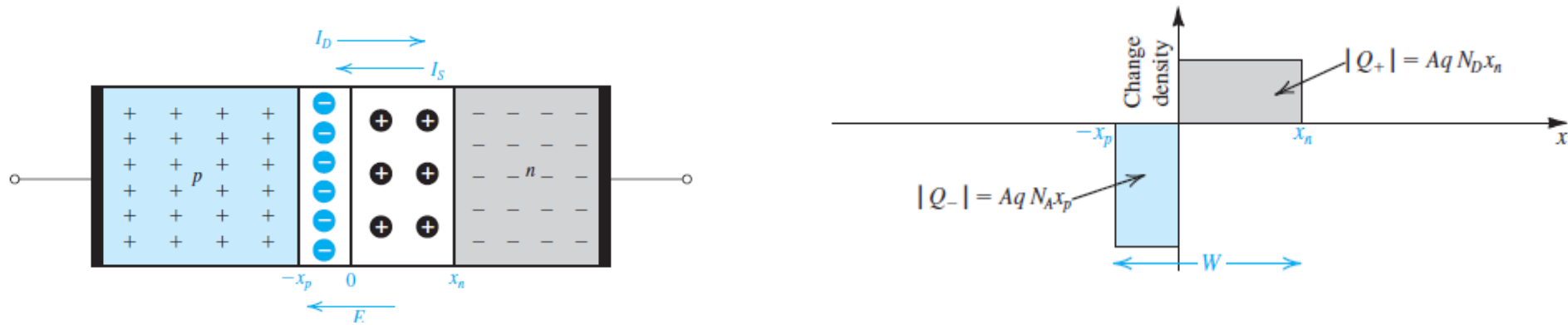
Typically W is in the range $0.1 \mu\text{m}$ to $1 \mu\text{m}$.

x_n and x_p can be rewritten as:

$$x_n = W \frac{N_A}{N_A + N_D}$$

$$x_p = W \frac{N_D}{N_A + N_D}$$

A largura da camada de depleção (W)



The charge stored on either side of the depletion region can be expressed in terms of W by:

$$Q_J = |Q_+| = |Q_-| \quad \longrightarrow \quad Q_J = Aq \left(\frac{N_A N_D}{N_A + N_D} \right) W \quad \longrightarrow \quad Q_J = A \sqrt{2\epsilon_s q \left(\frac{N_A N_D}{N_A + N_D} \right) V_0}$$

Example 4:

Consider a pn junction in equilibrium at room temperature ($T = 300$ K) for which the doping concentrations are $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$ and the cross-sectional area $A = 10^{-4} \text{ cm}^2$. Calculate p_p , n_{p0} , n_n , p_{n0} , V_0 , W , x_n , x_p , and Q_J . Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

Solution



$$p_p \approx N_A = 10^{18} \text{ cm}^{-3}$$



$$n_{p0} = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2 \text{ cm}^{-3}$$



$$n_n \approx N_D = 10^{16} \text{ cm}^{-3}$$



$$p_{n0} = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Consider a pn junction in equilibrium at room temperature ($T = 300$ K) for which the doping concentrations are $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$ and the cross-sectional area $A = 10^{-4} \text{ cm}^2$. Calculate p_p , n_{p0} , n_n , p_{n0} , V_0 , W , x_n , x_p , and Q_J . Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$V_T = \frac{kT}{q} = \frac{8.62 \times 10^{-5} \times 300 \text{ (eV)}}{q \text{ (e)}} \\ = 25.9 \times 10^{-3} \text{ V}$$

$$V_0 = 25.9 \times 10^{-3} \ln\left(\frac{10^{18} \times 10^{16}}{2.25 \times 10^{20}}\right) \\ = 0.814 \text{ V}$$

Consider a pn junction in equilibrium at room temperature ($T = 300$ K) for which the doping concentrations are $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$ and the cross-sectional area $A = 10^{-4} \text{ cm}^2$. Calculate p_p , n_{p0} , n_n , p_{n0} , V_0 , W , x_n , x_p , and Q_J . Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

$$\bullet \quad W = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) V_0} \longrightarrow W = \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}} \left(\frac{1}{10^{18}} + \frac{1}{10^{16}} \right) \times 0.814}$$

$$= 3.27 \times 10^{-5} \text{ cm} = 0.327 \text{ } \mu\text{m}$$

$$\bullet \quad x_n = W \frac{N_A}{N_A + N_D}$$

$$= 0.327 \frac{10^{18}}{10^{18} + 10^{16}} = 0.324 \text{ } \mu\text{m}$$

$$x_p = W \frac{N_D}{N_A + N_D}$$

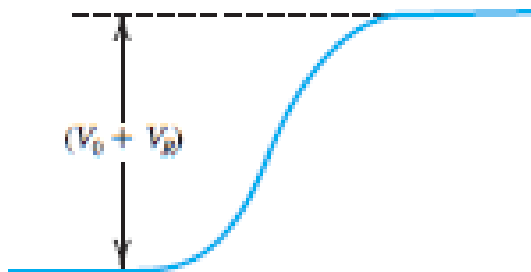
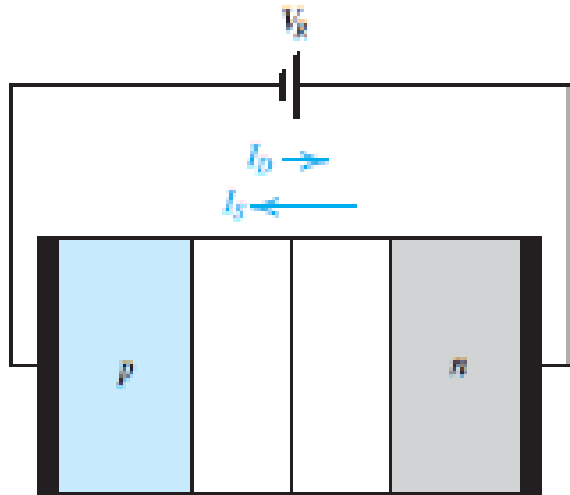
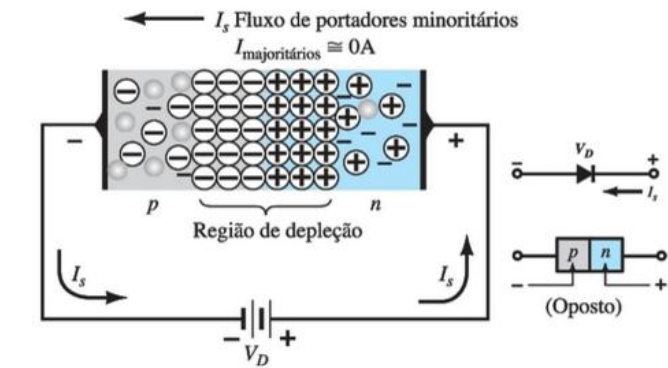
$$= 0.327 \frac{10^{16}}{10^{18} + 10^{16}} = 0.003 \text{ } \mu\text{m}$$

Consider a pn junction in equilibrium at room temperature ($T = 300$ K) for which the doping concentrations are $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$ and the cross-sectional area $A = 10^{-4} \text{ cm}^2$. Calculate p_p , n_{p0} , n_n , p_{n0} , V_0 , W , x_n , x_p , and Q_J . Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

● $Q_J = Aq \left(\frac{N_A N_D}{N_A + N_D} \right) W \longrightarrow Q_J = 10^{-4} \times 1.6 \times 10^{-19} \left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}} \right) \times 0.327 \times 10^{-4}$
 $= 5.18 \times 10^{-12} \text{ C} = 5.18 \text{ pC}$

**pn junction
with applied voltage**

**pn junction
reversed biased**



(b) Reverse Bias

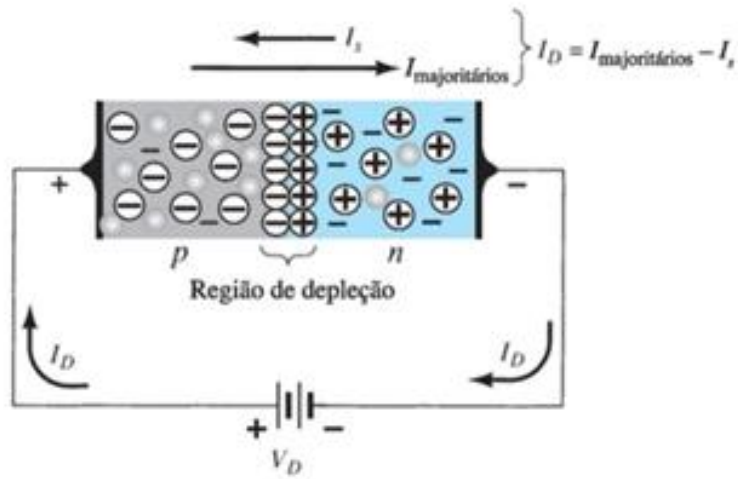
The externally applied reverse-bias voltage is in the direction to add to the barrier voltage, and it does, thus **increasing the effective barrier voltage to $(V_0 + V_R)$** as shown in the equations below.

This reduces the number of holes that diffuse into the n region and the number of electrons that diffuse into the p region. The end result is that the diffusion current is dramatically reduced. A reverse-bias voltage of a volt or so is sufficient to cause $I_D=0$, and the current across the junction and through the external circuit will be equal to I_s .

$$W = x_n + x_p = \sqrt{\frac{2\epsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (V_0 + V_R)}$$

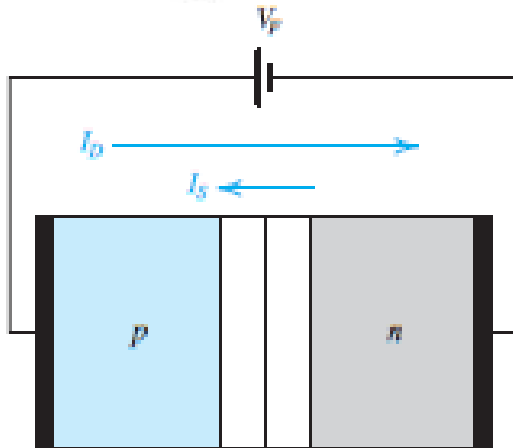
$$Q_J = A \sqrt{2\epsilon_s q \left(\frac{N_A N_D}{N_A + N_D} \right) (V_0 + V_R)}$$

**pn junction
forward biased**

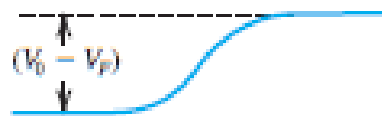


Here the applied voltage is in the direction that subtracts from the built-in voltage, resulting in a **reduced barrier voltage across the depletion region**.

Most importantly, the lowering of the barrier voltage will **enable more holes to diffuse from p to n** and **more electrons to diffuse from n to p**. Thus the diffusion current increases substantially and, as will be seen shortly, can become many orders of magnitude larger than the drift current I_s .



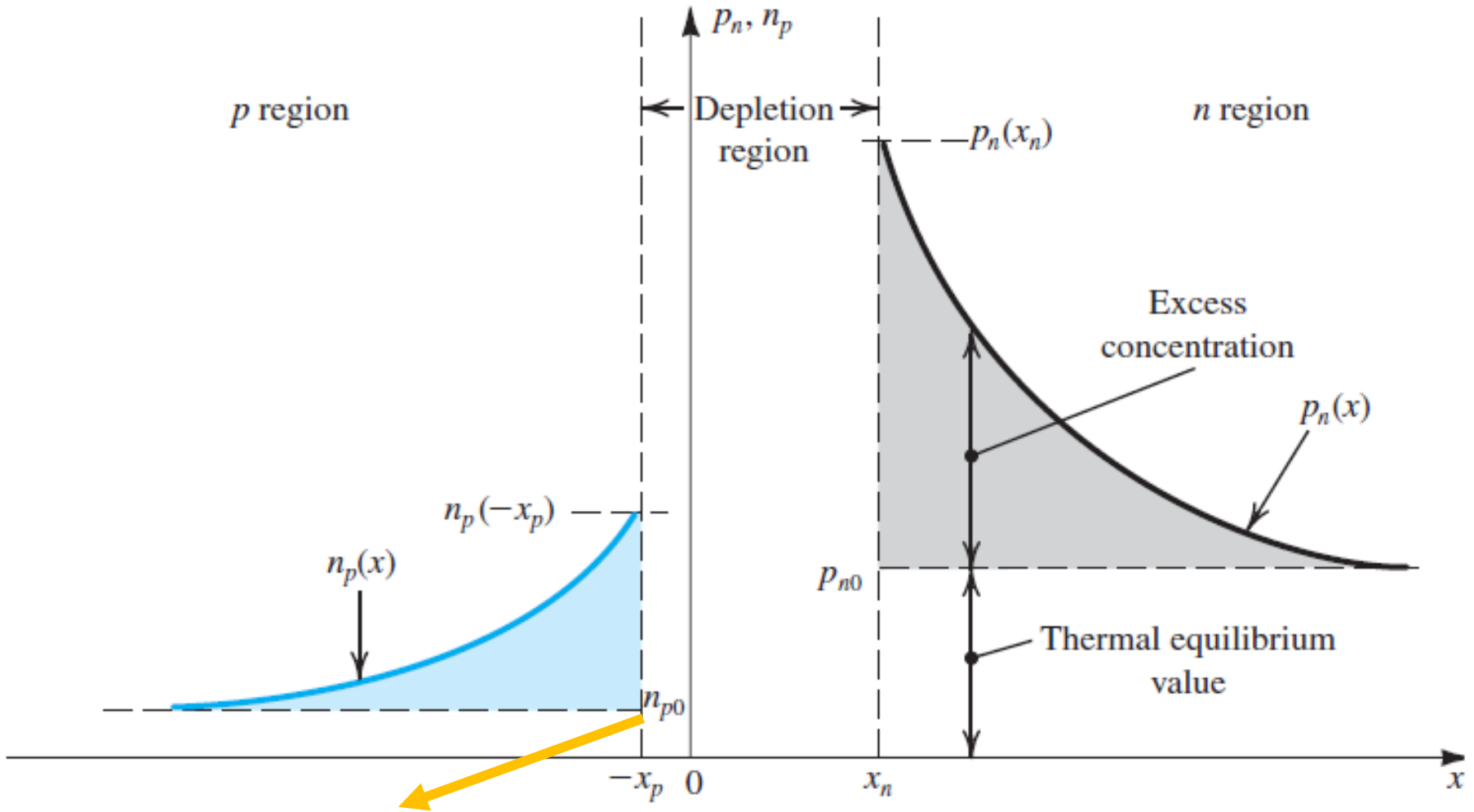
$$I = I_D - I_s$$



(c) Forward Bias

**The current-voltage
relationship
(Shockley's Equation)**

Minority carrier Distribution ($p_n(x)$ and $n_p(x)$) in a forward-biased junction ($N_A \gg N_D$)



$n_{p0} \ll p_{n0}$ if $N_A \gg N_D$ (as shown in example 3.4)

- The concentration of holes in the n region at the edge of the depletion region will increase considerably. In fact, an important result from device physics shows that the steady-state concentration at the edge of the depletion region will be:

$$p_n(x_n) = p_{n0} e^{V/V_T} \quad (\text{If } V=0, p_n(x_n)=p_{n0})$$

- The forward-bias voltage V results in an **excess concentration** of minority holes given by:

$$\text{Excess concentration} = p_{n0} e^{V/V_T} - p_{n0} = p_{n0} (e^{V/V_T} - 1)$$

- The total hole concentration in the n material will be given by:

$$p_n(x) = p_{n0} + (\text{Excess concentration}) e^{-(x-x_n)/L_p} = p_{n0} + p_{n0} (e^{V/V_T} - 1) e^{-(x-x_n)/L_p}$$

The exponential decay is characterized by the constant L_p , which is called the **diffusion length of holes in the n material**. The smaller the value of L_p , the faster the injected holes will recombine with the majority electrons, resulting in a steeper decay of minority carrier concentration.



$$p_n(x) = p_{n0} + p_{n0}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$

$$J_p(x) = -qD_p \frac{dp_n(x)}{dx}$$



$$J_p(x) = q\left(\frac{D_p}{L_p}\right)p_{n0}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$

If $x=x_n$



$$J_p(x_n) = q\left(\frac{D_p}{L_p}\right)p_{n0}(e^{V/V_T} - 1)$$



An exactly parallel development can be applied to the electrons that are injected from the n to the p region, resulting in an electron diffusion current given by:

$$J_n(-x_p) = q\left(\frac{D_n}{L_n}\right)n_{p0}(e^{V/V_T} - 1)$$

$$\longrightarrow I = A(J_p + J_n) = Aq\left(\frac{D_p}{L_p} p_{n0} + \frac{D_n}{L_n} n_{p0}\right)(e^{V/V_T} - 1)$$

■ Substituting for $p_{n0} = n_i^2 / N_D$ and $n_{p0} = n_i^2 / N_A$ gives:

$$I = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)(e^{V/V_T} - 1)$$

$$I = I_S(e^{V/V_T} - 1)$$

Shockley's Equation !

$$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$$

Example 5:

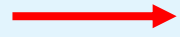
For the pn junction considered before $N_A = 10^{18}/\text{cm}^3$, $N_D = 10^{16}/\text{cm}^3$, $A = 10^{-4}\text{cm}^2$, $n_i = 1.5 \times 10^{10}/\text{cm}^3$, let $L_p = 5 \mu\text{m}$, $L_n = 10 \mu\text{m}$, D_p (in the n region) $= 10 \text{ cm}^2/\text{V}\cdot\text{s}$, and D_n (in the p region) $= 18 \text{ cm}^2/\text{V}\cdot\text{s}$. The pn junction is forward biased and conducting a current $I = 0.1 \text{ mA}$. Calculate: (a) I_S ; (b) the forward-bias voltage V ; and (c) the component of the current I due to hole injection and that due to electron injection across the junction.

$$\bullet \quad I_S = Aqn_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right) \longrightarrow I_S = 10^{-4} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2 \times \left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}} \right) = 7.3 \times 10^{-15} \text{ A}$$

(baixo valor !)

$$I = I_S(e^{V/V_T} - 1)$$

$$\approx I_S e^{V/V_T}$$



$$V = V_T \ln\left(\frac{I}{I_S}\right)$$

$$V = 25.9 \times 10^{-3} \ln\left(\frac{0.1 \times 10^{-3}}{7.3 \times 10^{-15}}\right)$$
$$= 0.605 \text{ V}$$

$$J_p(x) = q\left(\frac{D_p}{L_p}\right)p_{n0}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$



$$I_p = Aq\frac{D_p}{L_p}p_{n0}(e^{V/V_T} - 1)$$

$$= Aq\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{V/V_T} - 1)$$

Similarly I_n can be found using:

$$J_n(-x_p) = q\left(\frac{D_n}{L_n}\right)n_{p0}(e^{V/V_T} - 1)$$



$$I_n = Aq\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{V/V_T} - 1)$$



$$\frac{I_p}{I_n} = \left(\frac{D_p}{D_n}\right)\left(\frac{L_n}{L_p}\right)\left(\frac{N_A}{N_D}\right) = \frac{10}{18} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}} = 1.11 \times 10^2 = 111$$



$$I = I_p + I_n$$



$$I_p = \frac{111}{112} \times 0.1 = 0.0991 \text{ mA}$$

$$I_n = \frac{1}{112} \times 0.1 = 0.0009 \text{ mA}$$

Capacitive Effects in pn Junctions

Depletion or Junction Capacitance

When a *pn* junction is **reverse biased** with a voltage V_R , the charge stored on either side of the depletion region is given by:

$$Q_J = A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D} (V_0 + V_R)} \quad \longrightarrow \quad Q_J = \alpha \sqrt{V_0 + V_R}$$
$$\alpha = A \sqrt{2\epsilon_s q \frac{N_A N_D}{N_A + N_D}}$$

A **depletion or junction capacitance (C_j)** is defined as:

$$C_j = \left. \frac{dQ_J}{dV_R} \right|_{V_R=V_0} \quad \longrightarrow \quad C_j = \frac{\alpha}{2\sqrt{V_0 + V_R}}$$

The value of at zero reverse-bias can be obtained:

$$C_{j0} = \frac{\alpha}{2\sqrt{V_0}}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

$$C_{j0} = A \sqrt{\left(\frac{\epsilon_s q}{2}\right) \left(\frac{N_A N_D}{N_A + N_D}\right) \left(\frac{1}{V_0}\right)}$$

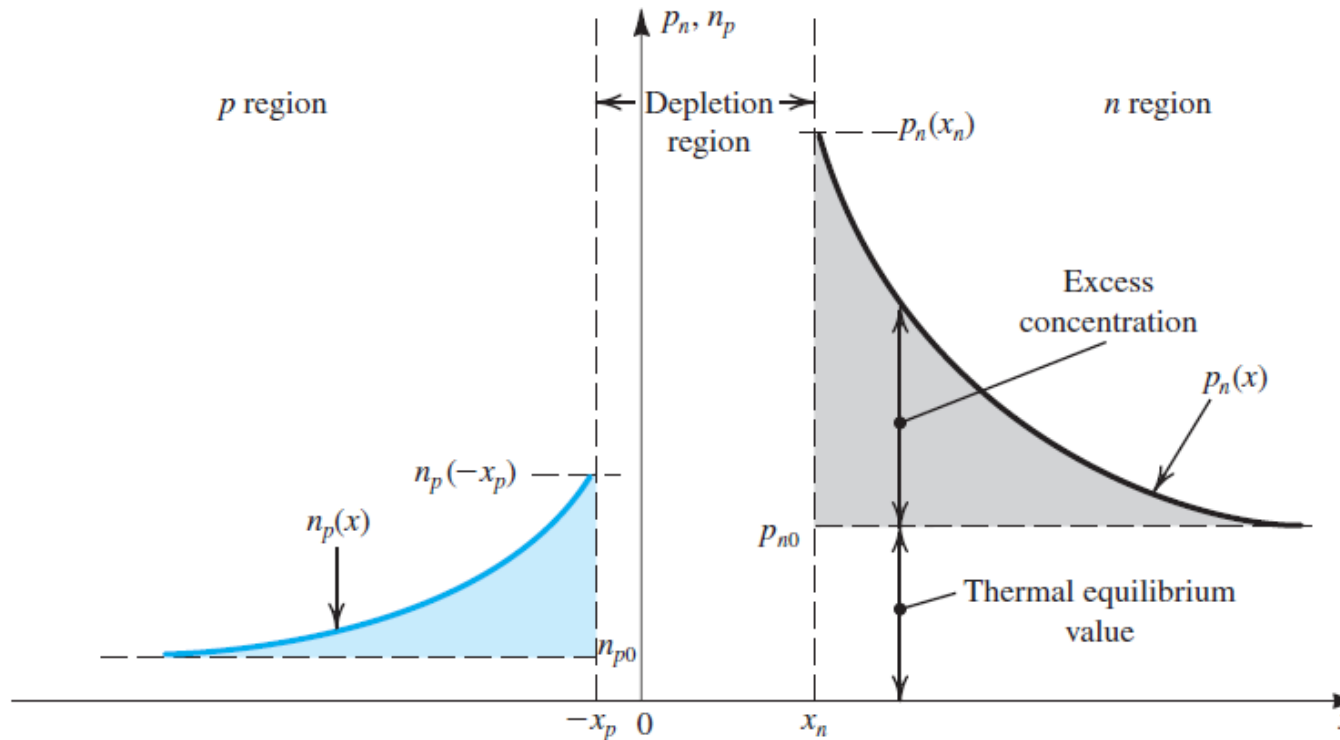
- We point out that in the pn junction we have been studying, the doping concentration is made to change abruptly at the junction boundary. Such a junction is known as an **abrupt junction**.
- There is another type of pn junction in which the carrier concentration is made to change gradually from one side of the junction to the other. To allow for such a **graded junction**.

$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_0}\right)^m}$$

$1/3 < m < 1/2$, depending on the manner in which the concentration changes from the p to the n side.

Diffusion Capacitance

Consider a **forward-biased pn junction**. In steady-state, minority carrier distributions in the p and n materials are established, as shown in figure below. Thus a certain amount of **excess minority carrier charge is stored in each of the p and n bulk regions (outside the depletion region)**. This charge-storage phenomenon gives rise to another capacitive effect.



- The excess hole charge stored in the n region can be found from the shaded area under the exponential as follows:

$$Q_p = Aq \times \text{shaded area under the } p_n(x) \text{ curve}$$

- It is shown that:

$$Q_p = Aq[p_n(x_n) - p_{n0}]L_p = \frac{L_p^2}{D_p} I_p$$

$$\tau_p = \frac{L_p^2}{D_p} \longrightarrow Q_p = \tau_p I_p$$

The time constant τ_p is known as the excess **minority carrier (hole) lifetime**. *It is the average time it takes for a hole injected into the n region to recombine with a majority electron.*

A relationship similar can be developed for the electron charge stored in the p region:

$$Q_n = \tau_n I_n$$

The time constant τ_n is known as the excess **minority carrier (electron) lifetime**. *It is the average time it takes for a electron injected into the p region to recombine with a majority hole.*

This charge can be expressed in terms of the diode current $I = I_p + i_n$ as:

$$Q = \tau_T I$$

The time constant τ_T is called called the **mean transit time** of the junction.

If $N_A \gg N_D$ it can be shown that $\tau_T \approx \tau_p$

■ For small changes around a bias point, we can define an **incremental diffusion capacitance (C_d)** as:

$$C_d = \frac{dQ}{dV}$$

It can be shown that:

$$C_d = \left(\frac{\tau_T}{V_T} \right) I$$

where **I is the forward-bias current**. Note that is directly proportional to the forward current I and thus C_d is negligibly small when the diode is reverse biased.